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COMBINED CONVECTION IN AN AXISYMMETRIC STAGNATION FLOW OF MICROPOLAR FLUID

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ABSTRACT

An analysis is presented to study the effects of buoyancy on forced convection in an axisymmetric stagnation flow of micropolar fluids over a vertical cylinder with constant or linear variation of surface heat flux conditions. Numerical solutions are given for the governing momentum, angular momentum and energy equations. Two flow regions, namely the buoyancy-assisted and buoyancy-opposed cases are analysed. It is observed that the wall shear stress and surface heat transfer rate increase or decrease with the buoyancy force parameter depending on the flow regime being buoyancy-assisted or buoyancy-opposed respectively.

KEY WORDS Convection Fluid flow Heat transfer Micropolar fluids Stagnation flow

NOMENCLATURE

A	= Constant used in eq. (6)
a	= Radius of cylinder
B	= Dimensionless material parameter
f	= Velocity profile function
G	= Dimensionless microrotation
Gr^*	= Modified Grashoff number
h	= Heat transfer coefficient
j	= Microinertia per unit mass
k	= Thermal conductivity
n	= Constant exponent
N	= Microrotation
Nu_z	= Local Nusselt number (hz/k)
Pr	= Prandtl number
p	= Pressure
q_w	= Heat flux at the wall
Re	= Reynolds number ($Aa^2/2\nu$)
r	= Co-ordinate normal to the cylindrical surface
T	= Temperature
u	= Velocity component in r-direction
w	= Velocity component in z-direction
z	= Co-ordinate parallel to the wall
η	= Dimensionless co-ordinate
β	= Volumetric expansion coefficient
ξ	= Buoyancy parameter
Θ	= Dimensionless temperature
μ	= Dynamic viscosity
κ	= Cross viscosity
ν	= Kinematic viscosity
ρ	= Fluid density
σ	= Dimensionless wall couple stress
$\lambda, \Delta, \varepsilon$	= Demensionless material parameters

Subscripts

w	= Surface conditions
∞	= Conditions far away from the surface

INTRODUCTION

Eringen[1] has developed the theory of micropolar fluids which show microrotation effects as well as microinertia. The theory may be applied to explain the flow of colloidal fluids, liquid crystals, fluids with additives, animal blood, etc.

The problem of finding exact solutions of the Navier-Stokes equations presents insurmountable mathematical difficulties. This is primarily due to the fact that the Navier-Stokes

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equations are non-linear. However, it is possible to find exact solutions in certain particular cases. Wang[2] presented an exact solution for the axisymmetric stagnation flow on an infinite cylinder. One of the present authors[3] provided the solutions corresponding to the steady state heat transfer in an axisymmetric stagnation flow over an infinite circular cylinder. Solutions for the temperature field were obtained for isothermal and uniform heat flux wall conditions for a wide range of Prandtl numbers and Reynolds numbers. Recently, Gorla *et al.*[4,5] examined the fluid flow and heat transfer characteristics in an oscillating laminar boundary layer in the vicinity of an axisymmetric stagnation point by means of a boundary layer approximation. They evaluated the amplitude and phase angle of the wall skin friction as well as heat transfer rate fluctuations for a wide range of the reduced frequency of oscillation.

A situation where both the forced and free convection effects are of comparable order is called mixed convection. In such a flow, the flow and thermal fields are no longer symmetric with respect to the stagnation line. The friction factor and local heat transfer rate can be quite different under these conditions relative to the forced convection case. Gorla[6] has studied the mixed convection in an axisymmetric flow on a vertical non-isothermal cylinder.

The inadequacy of the classical continuum approach to describe the mechanics of complex fluids has led to the development of theories of microcontinua in which continuous media are now regarded as sets of structured particles possessing not only mass and velocity but also a structure with which is associated a moment of inertia density and a microdeformation tensor. This extension of fluid mechanics required a complete reappraisal of classical concepts, viz the symmetry of the stress tensor, the absence of couple stresses, etc., in order to account for local structural aspects and micromotions. In fact, while many of the principles of classical continuum mechanics remain valid for this new class of fluids, they had to be augmented with additional balance laws and constitutive relations. The presence of microscopic elements in a fluid gives rise not only to classical Cauchy stresses but also to couple stresses due to the microelement interactions.

The earliest formulation of a general theory of fluid microcontinua is attributed to Eringen[1] in which the mechanics of fluids with deformable microelements are considered. The author developed a physical model in which each continuum particle is assigned a substructure, i.e. each material volume element contains microvolume elements which can translate, rotate and deform independently of the motion of the macrovolume; however, each deformation of the macrovolume element can be expected to produce a subsequent deformation of the microvolume elements. Thus a mechanism is provided in the theory to treat materials which are capable of supporting local stress moments and body moments and, in addition, are influenced by the microelement spin inertia. The micropolar theory was later on extended by Eringen[7] to take into account thermal effects, and has been termed the theory of thermomicrofluids, which represents the most general theory of micromorphic thermofluids.

In the present paper, we have presented an analysis for the forced and free convection of a micropolar fluid in the vicinity of an axisymmetric stagnation point on a vertical cylinder with a constant or linear variation of the surface heat flux. Numerical results are presented for a range of values of the material parameters, the buoyancy parameter and Prandtl number of the fluid.

ANALYSIS

Let us consider a steady, laminar, incompressible flow of a micropolar fluid at an axisymmetric stagnation point on an infinite cylinder. A model of the flow is shown in *Figure 1*. The flow is axisymmetric about the z-axis and also symmetric to the $z = 0$ plane. The stagnation line is at $z = 0$ and $r = a$. The temperature of the free stream fluid is taken as T_∞. The surface of the body is assumed to be subjected to q_w. The governing equations are given by:

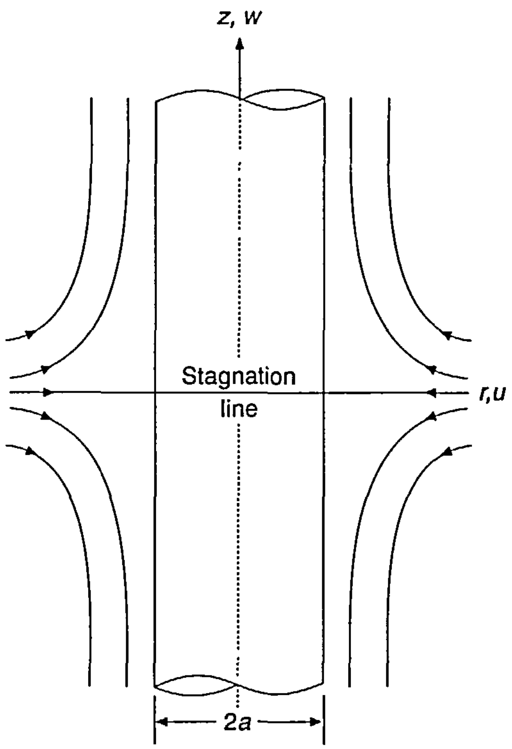

Figure 1 Flow model and co-ordinate system

Mass:

$$r\frac{\partial w}{\partial z}+\frac{\partial}{\partial r}(ru)=0 \tag{1}$$

Momentum:

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{1}{\rho}(\mu+\kappa)\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial z^2}\right]-\frac{\kappa}{\rho}\frac{\partial N}{\partial z} \tag{2}$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{1}{\rho}(\mu+\kappa)\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right]\pm g\beta(T-T_\infty)+\frac{\kappa}{\rho}\left(\frac{\partial N}{\partial r}+\frac{N}{r}\right) \tag{3}$$

Angular momentum:

$$\rho j\left[u\frac{\partial N}{\partial r}+w\frac{\partial N}{\partial z}\right]=\gamma.\left[\frac{\partial^2 N}{\partial r^2}+\frac{1}{r}\frac{\partial N}{\partial r}-\frac{N}{r^2}+\frac{\partial^2 N}{\partial z^2}\right]+\kappa\left[\frac{\partial u}{\partial z}-\frac{\partial w}{\partial r}\right]-2\kappa N \tag{4}$$

Energy:

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right] \tag{5}$$

The boundary conditions for the velocity and angular velocity field are given by

$$\begin{aligned} r=a: &\quad u=w=0, \quad N=-\frac{1}{2}\left(\frac{\partial w}{\partial r}\right) \\ r\to\infty: &\quad u=-A\left(r-\frac{a^2}{r}\right), \quad w=2Az, \quad N\to 0 \end{aligned} \tag{6}$$

For the temperature field we have

$$\begin{aligned} r = a: &\quad \frac{\partial T}{\partial r} = -\frac{q_w}{k_f} \\ r \to \infty: &\quad T \to T_\infty \end{aligned} \tag{7}$$

In (3) the density variation is taken into account by the Boussinesq approximation. The term which represents the buoyancy force effect on the flow field has ± signs. The plus sign indicates buoyancy-assisted flow whereas the negative sign stands for buoyancy-opposed flow.

We now let

$$\begin{aligned} \eta &= \left(\frac{r}{a}\right)^2 \\ \xi &= \frac{Gr_z^*}{\mathrm{Re}_z^{\frac{5}{2}}} \\ u &= -Aa\eta^{-1/2}.f(\eta) \\ w &= 2Af'(\eta)z \\ N &= 4\frac{A}{a}.z.\eta^{1/2}.G(\eta) \\ \theta &= \frac{T-T_\infty}{\left(\frac{q_w a}{2k_f}\right)} \\ Gr_z^* &= \frac{g\beta q_w z^4}{K_f v^2} \\ \mathrm{Re} &= \frac{Aa^2}{2v} \\ \mathrm{Re}_z &= \frac{Az^2}{2v} \\ \Delta &= \frac{\kappa}{\mu} \\ \lambda &= \frac{\gamma}{\mu j} \\ B &= \frac{a^2}{j} \end{aligned} \tag{8}$$

It may be verified that the continuity equation is satisfied. Upon substituting the expressions in (8) into equations (2-5) we have

$$(1+\Delta)(\eta f''' + f'') + Re[1 + ff'' - (f'^2)] + [\eta G' + G] \pm \frac{Re^{\frac{3}{2}}}{32}\xi\theta = 0 \tag{9}$$

$$\lambda[\eta^2 G'' + 2\eta G'] + Re[f(\frac{G}{2} + \eta G') - \eta f'G] - \Delta . B[\frac{\eta f''}{2} + \eta G] = 0 \tag{10}$$

$$\frac{1}{Pr}[\eta\theta'' + \theta'] + Ref\theta' = \frac{Ref'\theta z}{q_w}\frac{dq_w}{dZ} - \frac{\alpha}{4}\frac{a^2}{v}\frac{\theta}{q_w}\frac{d^2 q_w}{dZ^2} \tag{11}$$

In the above equations, a prime indicates differentiation with respect to η only. We note that similarity solutions exist if $d^2q_w/dz^2 = 0$. Assuming $q_w = cz^n$, we have $n = 0$ or 1 in order for separation of variables to be satisfied. Under these conditions, (11) becomes

$$\frac{1}{Pr}[\eta\theta'' + \theta'] + Ref\theta' - nRef'\theta = 0. \tag{12}$$

The transformed boundary conditions are given by

$$\begin{aligned} &f(\xi,1) = f'(\xi,1) = 0, \quad \theta'(\xi,1) = -1, \quad G(\xi,1) = -\frac{1}{2}f''(\xi,1) \\ &f'(\xi,\infty) = 1, \quad \theta'(\xi,\infty) = 0, \quad G(\xi,\infty) = 0. \end{aligned} \tag{13}$$

The buoyancy parameter ξ is inversely proportional to z when n = 0 and is independent of z when n = 1. When $(g\beta C/k_f v^2)(2v/A)^{5/2} = 1$ along with n = 1, the buoyancy parameter ξ = 1 and the situation will be called as the self-similar case for the linear wall heat flux. This condition is one of many possibilities for the linear wall heat flux conditions that may be prescribed. The wall shear stress may be written as

$$\begin{aligned} \tau_w &= (\mu + \kappa)\frac{\partial w}{\partial r})_{r=a} + \kappa N)_{r=a} \\ &= (\mu + \frac{\kappa}{2})\frac{\partial w}{\partial r})_{r=a} \end{aligned} \tag{14}$$

The friction factor is given by

$$C_{f_z} = (Re\,.\,Re_z)^{-\frac{1}{2}}[(1 + \frac{\Delta}{2})\,f''(\xi,1)] \tag{15}$$

The wall couple stress may be written as

$$M_w = \sqrt{Re\,.\,Re_z}\,\frac{8v\gamma}{a^3}[\frac{(\varepsilon - 1)}{2}f''(\xi,1) + 2G'(\xi,1)] \tag{16}$$

where $\varepsilon = (\beta/\gamma)$.

The dimensionless wall couple stress σ becomes

$$\sigma = \frac{M_w}{\rho A^2 z^3} = \frac{2\lambda}{B}.(Re_z)^{-1}[\frac{(\varepsilon - 1)}{2}f''(\xi,1) + 2G'(\xi,1)] \tag{17}$$

The local Nusselt number is given by

$$Nu_z = \frac{2\left[\frac{Re_z}{Re}\right]^{\frac{1}{2}}}{\theta(\xi,1)} \tag{18}$$

Equations (9), (10) and (12) are solved numerically for the cases n = 0 and n = 1 for both buoyancy-assisted and buoyancy-opposed flow regions. We note that n = 0 corresponds to uniform surface heat flux condition whereas n = 1 represents the linear variation of surface heat flux.

RESULTS AND DISCUSSION

The numerical procedure used here solves the two-point boundary value problems for a system of N ordinary differential equations in the range (X, X_1). The system is written as and the derivatives

$$\frac{dy_i}{dx} = f_i(x, y_1, y_2, \ldots, y_N) \quad i = 1, 2, \ldots, N$$

are evaluated by a procedure that evaluates the derivatives of Y_1, Y_2, ..., Y_N at a general point x. Initially N boundary values of the variable y_i must be specified, some of which will be specified at x and some at x_1. The remaining N boundary values are guessed and the procedure corrects them by a form of Newtonian iteration. Starting from the known and guessed values of y_i at x,the procedure integrates the equations forward to a matching point R, Using Merson's method. Similarly, starting from x_1 it integrates backwards to R. The difference between the forward and backward values of y_t at R should be zero for a true solution. The procedure uses a generalized Newton method to reduce these differences to zero, by calculating corrections to the estimated boundary values. This process is repeated iteratively until convergence is obtained to a specified accuracy. The tests for convergence and the perturbation of the boundary conditions are carried out in a mixed form, e.g., if the error estimate for y_t is ERROR$_i$, we test whether ABS (ERROR$_i$) < ERRORi × (1 + ABS y_i).

A solution was considered to be converged if the newly calculated values of f, f', G, G' and θ differed from their previous guessed values within a tolerance of $\in \leq 10^{-5}$. The numerical results were found to depend on η_∞ and the step size $\Delta\eta$. We have used $\Delta\eta = 0.005$ and $\eta_\infty = 20$ without causing numerical oscillations in the values of f, f', G, G, θ and θ'.

Table 1 Local friction factor, wall couple stress and Nusselt number for n = 1 and n = 0 respectively

			Buoyancy-assisted flow			Buoyancy-opposed flow		
Pr	λ	Δ	$F''(\xi, 1)$	$G'(\xi, 1)$	$1/\Theta'(\xi, 1)$	$F''(\xi, 1)$	$G'(\xi, 1)$	$1/\Theta'(\xi, 1)$
10	0.5	0.5	10.52955	0.026446	18.94823	10.41253	0.02639	18.89607
			10.54257	0.026466	12.91243	10.40013	0.02638	12.88450
		5.0	5.146001	0.193973	15.00676	5.107052	0.19370	14.98084
			5.150034	0.194023	10.7506	5.102448	0.19365	10.73321
		50	1.700126	1.058541	10.61145	1.693143	1.05774	10.60242
			1.700901	1.058711	7.857541	1.692365	1.05758	7.850362
	5.0	0.5	10.52894	0.003861	18.94789	10.41346	0.00386	18.89651
			10.54188	0.003861	12.91245	10.40049	0.00386	12.88449
		5.0	5.148862	0.032404	15.00777	5.105089	0.03238	14.98166
			5.152540	0.032408	10.75057	5.105188	0.03238	10.73327
		50	1.721074	0.218386	10.61815	1.713336	0.21832	10 60913
			1.721833	0.218400	7.854514	1.713357	0.21830	7.847397
	50	0.5	10.52897	0.000468	18.94791	10.41350	0.00047	18.89653
			10.54191	0.000468	12.91246	10.40052	0.00047	12.88450
		5.0	5.149785	0.004398	15.00837	5.110319	0.00439	14.98226
			5.153606	0.004398	10.75094	5.106255	0.00440	10.73363
		50	1.734967	0.037486	10.63777	1.728028	0.03748	10.62881
			1.735708	0.037487	7.866285	1.727293	0.03748	7.859265
100	0.5	0.5	10.48729	0.026421	54.20113	10.45493	0.02642	54.16299
			10.49926	0.026423	32.78130	10.44294	0.02641	32.71589
		5.0	5.133923	0.193862	38.4341	5.118740	0.19381	38.40368
			5.137129	0.193875	21.20029	5.115675	0.1938	21.17710
		50	1.698199	1.058242	23.52433	1.695071	1.05805	23.51316
			1.698635	1.058288	15.30443	1.694640	1.05801	15.29929
	5.0	0.5	10.48741	0.003859	54.20126	10.45504	0.00389	54.16311
			10.50032	0.003859	32.78277	10.44201	0.00386	32.71451
		5.0	5.136840	0.032396	38.43974	5.121585	0.03239	38.41001
			5.139981	0.032397	21.20343	5.117750	0.03239	21.17906
		50	1.719154	0.218359	23.59334	1.716035	0.21834	23.58224
			1.719607	0.218363	15.32277	1.715583	0.21834	15.31755
	50	0.5	10.48745	0.000468	54.20132	10.45508	0.00047	54.16317
			10.50035	0.000468	32.78284	10.44205	0.00047	32.71458
		5.0	5.137648	0.004398	38.44168	5.122492	0.00439	38.4133
			5.141048	0.004398	21.20508	5.118814	0.00440	21.18071
		50	1.73305	0.037485	23.6607	1.729945	0.03748	23.64906
			1.733519	0.037485	15.34742	1.729479	0.03748	15.34219

The buoyancy parameter ξ is inversely proportional to z when $n = 0$ and is independent of z when $n = 1$, as revealed by equation (8). For $n = 1$ and $C = [A^5 K_f^2/(g^2 \beta^2 \nu)]^{1/2}$ we obtain a similarity solution. This case was examined for buoyancy-assisted as well as buoyancy-opposed cases. Numerical results representing the friction factor, wall couple stress and Nusselt number are displayed in *Table 1* for a range of micropolar parameters. When $\Delta = 0$, the present results coincided with the Newtonian fluid data reported by Gorla[6]. These details are not reproduced here in the interest of conserving space.

As Δ increases we observe that the friction factor and the heat transfer rate decrease. This indicates that micropolar fluids possess drag reducing and heat transfer suppression characteristics. The wall couple stress is observed to increase with Δ. As the Prandtl number increases, the local Nusselt number increases for both buoyancy-assisting and opposing flows. *Figure 2* displays results for local wall shear stress. All computations were done from $\xi = 0$ to 10. Both cases, $n = 0$ and $n = 1$ were considered. In the case of the buoyancy-assisted flow regime, the buoyancy force increases the friction factor whereas in the case of buoyancy-opposed case, the opposite behaviour is observed. As $\xi \rightarrow 10$, the buoyancy-opposed case results in boundary layer separation. In the buoyancy-assisted flow region, the friction factor in the case of uniform surface heat flux case ($n = 0$) is higher than the linear variation of surface heat flux case ($n = 1$). The results indicated that local wall shear stress decreases as Δ increases for both buoyancy-assisting and -opposing flows. The numerical results also indicated that the Nusselt number decreases as Δ increases for both buoyancy-assisting and -opposing flows. *Figure 3* shows that the Nusselt number in the uniform surface heat flux case ($n = 0$) is less than the linear surface heat flux case ($n = 1$) in the buoyancy-assisted flow regime.The buoyancy-opposed flow regime displays the opposite behaviour. The Nusselt number increases with Prandtl number. *Figure 4* displays the distribution of wall couple stress within the boundary layer. The results indicated that as Δ increases, the wall couple stress increases.

It may be noted that the results presented in this paper are valid in a small region in the vicinity of the stagnation line. The increase in the buoyancy parameter is mainly due to an increase in the wall heat flux rather than an increase in the axial distance along the cylinder.

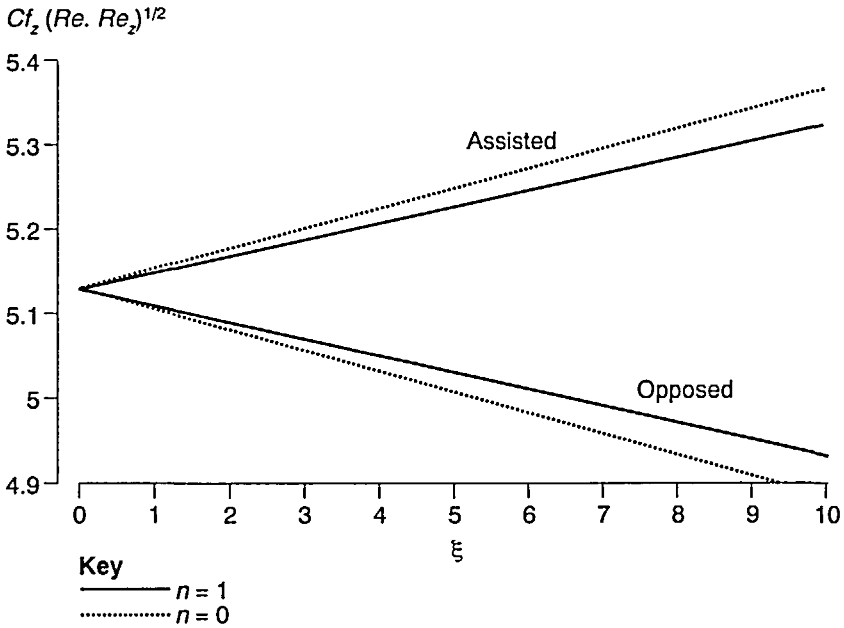

Figure 2 Local friction factor versus ξ ($\Delta = 5.0$)

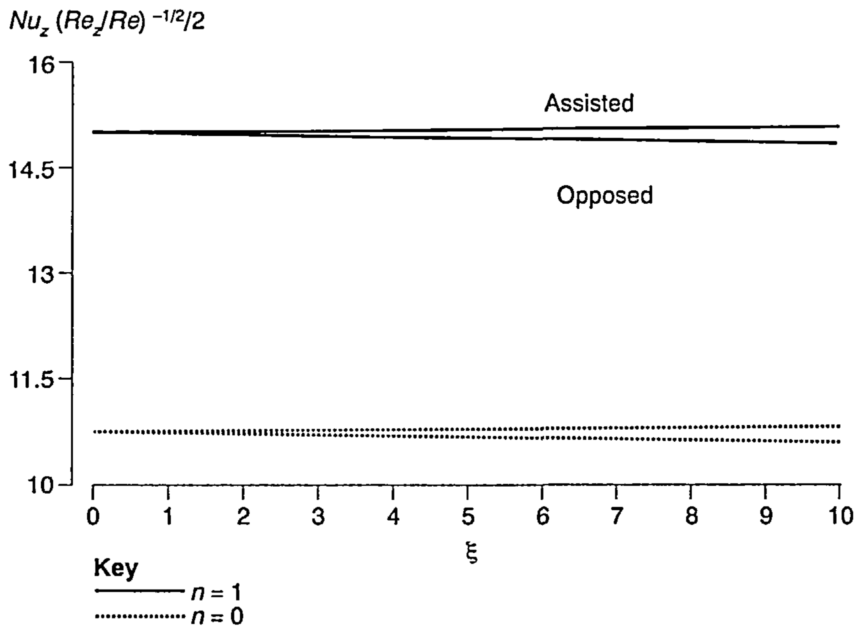

Figure 3 Local Nusselt number versus ξ ($\Delta = 5.0$)

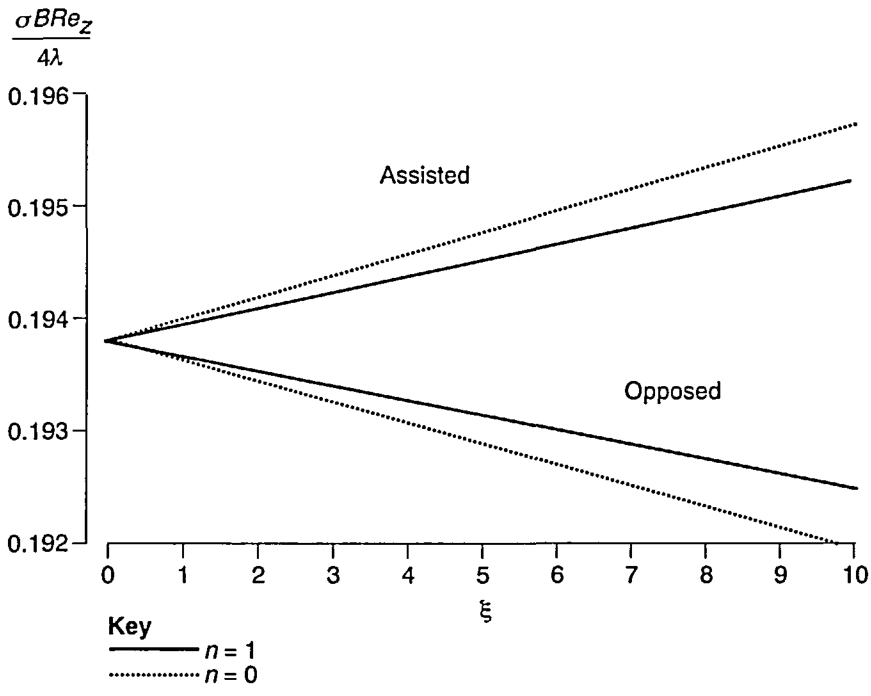

Figure 4 Local wall couple stress versus ξ ($\Delta = 5.0, \in = 1$)

CONCLUDING REMARKS

In this paper we have studied the combined forced and free convection in axisymmetric stagnation flows of micropolar fluids by using theory of micropolar fluids in the vicinity of a vertical heated cylinder. Numerical solutions are presented for the transformed equations governing the fluid flow and heat transfer. The missing wall values of the velocity, angular velocity and thermal functions are tabulated for a range of the dimensionless grouping of the material parameters.The analysis considered two flow regions, namely the buoyancy-assisted and buoyancy-opposed cases. In the case of the buoyancy-assisted regime, the friction factor and Nusselt number increase with increasing values of the buoyancy parameter, whereas, the opposite trend was observed to be true for the buoyancy-opposed flow regime. As the material parameter Δ increases, both friction factor and heat transfer rate decrease. This indicates that micropolar fluids reduce drag and heat transfer rate.

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